THE THEORY OF FLUCTUATIONS OF THE SURFACE BRIGHTNESS IN THE MILKY WAY

At present we can take as established that the absorbing layer in our Galaxy has a patchy structure, i.e., it consists of a large number of absorbing clouds whose cumulative effect is responsible for the general cosmic absorption. The absorbing clouds which are nearer to us and have higher absorbing power appear to us as *dark nebulae.* A large number of papers devoted to the study of dark nebulae has been published. However, a statistical study of the whole complex of absorbing clouds, which sometimes have low absorbing power, is still missing.

In an earlier paper [1], the author has shown that the fluctuations in the number of extragalactic nebulae are at least partly caused by the patchy structure of the galactic absorbing layer. In [1] some mean value of optical thickness of a cloud was deduced. It turned out to be equal to $0^m.27$.

We can adopt as a working hypothesis that the fluctuations of surface brightness along the Milky Way are also caused by absorbing clouds. Apparently such an assumption can be considered as the first approximation to the real situation. Then the question arises about the distribution law of the brightness fluctuations. In the following paragraphs we derive a differential equation which determines the distribution function. From the latter we find the values of moments of all orders.

1. Let us suppose that the equatorial plane of the Galaxy is homogeneously filled by stars and absorbing clouds up to an *infinite* distance.

This model assumption is not as bad as it seems, since absorption at very large distances is almost complete, and distant stars and clouds do not influence the observed brightnesses. At the same time we will assume that during the passage of light through each cloud the same fraction of the intensity is absorbed. The transparency of a cloud we will denote by q.

If the number of clouds in a certain direction at distances less than s is $n(s)$ (this is, of course, a random function), then the light of a star placed at distance s will diminish by the factor $q^{n(s)}$. Assume that an element dV

of galactic space emits total energy $4\pi\eta dV$. Then the observed brightness in any direction within the galactic plane will be

$$\int_0^\infty q^{n(s)}\,\eta\,ds.$$

We consider the distribution function of values of this integral:

$$f(I) = P\left(\int_0^\infty q^{n(s)}\,\eta\,ds > I\right).$$

For small values of $a > 0$ we can write

$$f(I) = P\left(\int_0^a q^{n(s)}\,\eta\,ds + q^{n(a)}\int_a^\infty q^{n(s)-n(a)}\eta\,ds > I\right). \qquad (1)$$

Since a is small, the number $n(a)$ can take on only two values, $n(a) = 0$ and $n(a) = 1$. The probability of 0 is $1 - \nu a$ and of 1 is νa. Here ν is the mean number of clouds per unit length of light path. The probabilities of other values of $n(s)$ are small numbers of higher orders and can be neglected.

Correspondingly, the integral $\int_0^a q^{n(s)}\,\eta\,ds$ can take on either a value ηa or $\eta\theta a$, where $0 < \theta < 1$. Therefore,

$$\begin{aligned} f(I) =& (1-\nu a)\,P\left(\int_a^\infty q^{n(s)-n(a)}\eta\,ds > I - a\eta\right) + \\ &+ \nu a\,P\left(\int_0^\infty q^{n(s)-n(a)}\eta\,ds > \frac{I-\eta\theta a}{q}\right). \end{aligned} \qquad (2)$$

But owing to homogeneous distribution of clouds in space

$$P\left(\int_a^\infty q^{n(s)-n(a)}\eta\,ds > I\right) = P\left(\int_0^\infty q^{n(s)}\,\eta\,ds > I\right),$$

and equation (2) can be rewritten in the form

$$f(I) = (1-\nu a)\,f(I-a\eta) + \nu a\,f\left(\frac{I-\eta\theta a}{q}\right). \qquad (3)$$

Up to the terms of the second order in a, this is equivalent to

$$f(I) = f(I) - \nu a\,f(I) - a\eta\,f'(I) + \nu a\,f\left(\frac{I}{q}\right).$$

From this

$$f(I) + \frac{\eta}{\nu} f'(I) = f\left(\frac{I}{q}\right) \tag{4}$$

or using a new variable $u = I\frac{\nu}{\eta}$

$$f(u) + f'(u) = f\left(\frac{u}{q}\right). \tag{5}$$

By differentiating, for the density $g(u) = f'(u)$ we obtain the equation

$$g(u) + g'(u) = \frac{1}{q}\, g\left(\frac{u}{q}\right). \tag{6}$$

From this functional equation it is possible to find the mean values of all powers of brightness.

From (5) we can see that

$$f'(0) = g(0) = 0.$$

Now multiplying (6) by u and integrating, we find

$$\overline{u} + \int_0^\infty g'(u)\, u\, du = q\overline{u},$$

where $\overline{u}$ is the mean brightness.

Integrating by parts and taking into account that

$$\int_0^\infty g(u)\, du = 1,$$

we find

$$\overline{u} = \frac{1}{1-q}. \tag{7}$$

Multiplying (6) by u^2 and integrating, we get

$$\overline{u^2}\,(1-q^2) = -\int_0^\infty g'(u)\, u^2\, du = 2\overline{u},$$

i.e., the mean value of the squared brightness equals

$$\overline{u^2} = \frac{2\overline{u}}{1-q^2} = \frac{2}{(1-q)^2(1+q)}.$$

For the relative square deviation, we obtain

$$\frac{\overline{(u-\overline{u})^2}}{\overline{u}^2} = \frac{\overline{u^2}}{\overline{u}^2} - 1 = \frac{1-q}{1+q}, \tag{8}$$

i.e., it is completely determined by the transparency of one cloud.

2. Now we abandon the assumption that all absorbing clouds have the same optical thickness. Rather let them have random optical thicknesses. However, we assume that in different parts of space the distribution of transparency q remains the same. Let the probability of having the value of transparency between q and $q + dq$ be $d\varphi(q)$.

In this case for the distribution function f of brightness, we obtain the following generalization of (5)

$$f(u) + f'(u) = \int_0^1 f(u/q)\, d\varphi(q). \tag{9}$$

For the probability density $g(u) = -f'(u)$, we get

$$g(u) + g'(u) = \int_0^1 g\left(\frac{u}{q}\right) \frac{d\varphi(q)}{q}. \tag{10}$$

The moments $\overline{u}$ and $\overline{u^2}$ are found to be

$$\overline{u} = \frac{1}{1 - \int_0^1 q\, d\varphi(q)} = \frac{1}{1 - \overline{q}}; \qquad \overline{u^2} = \frac{2\overline{u}}{1 - \int_0^1 q^2\, d\varphi(q)} = \frac{2\overline{u}}{1 - \overline{q^2}}, \tag{11}$$

where $\overline{q}$ and $\overline{q^2}$ are the mean values

$$\overline{q} = \int_0^1 q\, d\varphi(q); \qquad \overline{q^2} = \int_0^1 q^2\, d\varphi(q).$$

Since $\overline{q^2} > \overline{q}^2$, we have

$$\overline{u^2} > \frac{2\overline{u}}{1 - \overline{q}^2}.$$

It turns out that given $\overline{q}$, the relative mean square deviation

$$\frac{\overline{u^2} - \overline{u}^2}{\overline{u}^2} = \frac{\int_0^1 (1-q)^2\, d\varphi(q)}{\int_0^1 (1-q^2)\, d\varphi(q)}$$

is minimal in the case of clouds of identical transparency equal to $\overline{q}$.

For a number of simpler laws $\varphi(q)$, Laplace transforms of the solutions of equation (10) can be derived. In this way we find that when $\varphi(q) = q^k$, equation (10) has a solution $g(u) = u^k\, e^{-u} (k!)^{-1}$.

REFERENCES

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Yerevan
Astronomical Observatory of the Academy
of Sciences of Armenia